Calculating 7 Billion

Materials:

Part 1: Student worksheets
500 sheets, or 1 ream, of copy paper
1 24-pack of crayons (may be substituted for another geometrical object if desired)
Meter sticks for each group
U.S. Road Atlas
Computer and spreadsheet program such as Excel

Part 2: One napkin for every student
Napkin folding worksheet (one per student)

Part 3: Computer with ability to hook up to a projector
Projector
Interactive white board (optional)
Calculators with natural log function

Part 1: Measuring a Million

Procedure:

Divide the class into groups of 4-5 students. Each group will be responsible for doing one of the following five activities. Distribute the "Measuring a Million" Student Worksheets if you want your students to have the benefit of the suggested steps for each problem. If you prefer to have students devise their own steps, simply distribute the cards from page 3. When the activities are finished, have each group present its methods and findings to the other students, who should take notes on the presentations. Following the presentation, ask questions such as "Do you think that your answer is reasonable? How did your findings differ from your original estimates?"

Group #1: What would be the length of a million people holding hands? The length of a billion people holding hands?

(*Note: The calculations and answers may vary quite a bit due to the sizes of the people measured and whether their arms are down by their sides or stretched out. Have students include this information when showing their work.*)

Concept:

To understand the implications of world population at seven billion, it is important for students to be able to conceptualize and manipulate large numbers and to understand growth curves.

Objectives:

Students will be able to:
- Develop and implement a strategy for solving geometry problems in a cooperative learning group. (Part 1)
- Visualize doubling time through a group demonstration and math activity. (Part 2)
- Use technology to graph a scatter plot and estimate future population growth based on a current trend in simulation. (Part 3)

Subjects:

Economics, Math

Skills:

Calculating with large numbers, estimating, measuring length/area/volume, averaging, comparing the properties of linear, exponential, ploynomial and logarithmic functions, creating and using scatterplots, presenting methods for solving a complex problem.

Method:

Through cooperative learning activities and a class demonstration, students work through problems to visualize large numbers and use technology to graph population growth trends to make estimates about future growth.

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Group #2: How many people standing on each others' heads would it take to stack people to the moon? How tall would one billion people be if they were stacked in this way?

(*Note: The moon is 382,500 km from earth. The calculations and answers may vary quite a bit depending on the heights of the people measured. Have students include information on how they arrived at an average height when showing their work.*)

Group #3: How many sheets of copy paper could fit in the classroom? How many classrooms would you need to fit one billion sheets of paper?

(*Note: Provide students with a new ream of copy paper to measure. A ream includes 500 sheets of paper.*)

Group #4: How many 24-pack boxes of crayons could fit in the classroom? How many classrooms would you need to fit one billion crayon boxes?

(*Note: You can substitute other objects for crayon boxes of similar or different geometric shapes.*)

Group #5: Imagine a crowd of a million people. How big a field would you need to contain all of them? How much land would you need to hold a billion people?

(*Note: As with #1 and #2, the calculations will depend on the size of the people being measured. Have students include their methods when showing their work.*)

Follow-Up Activities:

1. There are over seven billion people in the world. What would be the length of a row of seven billion people?

2. Millions and billion can also be used for solving time problems. Will you still be living a million minutes from now? How about a billion minutes from now?

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To have your students come up with their own process for solving the problems, cut out the problems from this page and distribute one to each group.

Group 1:

A. What would be the length of a million people holding hands?

B. The length of a billion people holding hands?

Explain your methods and show your work.

Group 2:

A. How many people standing on each others heads would it take to stack people to the moon? The moon is 382,500 km from earth.

B. How tall would one billion people be if they were stacked in this way?

Explain your methods and show your work.

Group 3:

A. How many sheets of copy paper could fit in the classroom?

B. How many classrooms would you need to fit one billion pieces of paper?

Explain your methods and show your work.

Group 4:

A. How many boxes of crayons (a 24-pack) could fit in the classroom?

B. How many classrooms would you need to fit one billion crayon boxes?

Explain your methods and show your work.

Group 5:

A. Imagine a crowd of a million people. How big a field would you need to contain them all?

B. How much land would you need to hold a billion people?

Explain your methods and show your work.

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Part 2: Napkin Fold

Procedure:

1. Give each student a cocktail napkin or paper towel (regular paper is too thin).

2. Instruct them to fold the napkin in half, then in half again, in half a third time, and then in half a fourth time. At this point, it should be about 1 cm or 0.4 inches thick.

3. Ask them how thick the paper would be if you folded it in half 29 more times (if this were possible). Estimates may vary widely.

4. Tell them, "If we were to fold this napkin 29 more times, it would be 3,400 miles thick – the distance from Boston, Massachusetts, to Frankfurt, Germany." Explain that the napkin folding illustrates exponential growth, where you start doubling small numbers but are soon doubling ever larger numbers. Have students calculate the number of napkin layers after each fold and fill in the table on the Napkin Fold Student Worksheet.

5. Explain that the size of the human population has grown exponentially. Since the first human, we have doubled our population about 33 times. Our population is over 7 billion. Like in the napkin stretching from Boston to Frankfurt, each person on earth is like one layer of napkin.

Discussion Question:

1. Can our population continue to double like it has been? Why or why not?

   No, it will not continue to double because natural resources and space are limited. Already, population growth is beginning to plateau as we approach the carrying capacity of the Earth, or the maximum number of people the Earth can sustain.

Possible Extentions:

Have the students graph the first 5 data points on a graphing calculator or by hand. The equation for this exponential doubling time curve is y = 2X. Ask the students to use this equation and scientific calculators to solve the following questions about future population growth.

1. If we continued to double our population at this rate, how many times would we have to double to reach a population of 10 billion? 100 billion? (Hint: Set y equal to 10 billion or 100 billion and solve for x.)

2. What would the population be if we doubled 50 times? 100 times? (Hint: Set x equal to 50 or 100 and solve for y.)

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Part 3: Graphing Growth

* Note: This activity is designed for higher level math courses such as advanced mathematics, calculus, or statistics. It deals with graphing non-linear functions, using technology to find best fit curves, and solving exponential equations. The procedure calls for the teacher to demonstrate for the class, but an alternative would be for the students to explore this concept on their own in the school's computer lab using Excel. They could also use a graphing calculator or online tool to help them complete the task.

Procedure:

1. Hook up a projector to your computer and display your screen in front of the class so that all of the students can see what you are graphing. As you move through each step, explain what you are doing to the class.

2. Open an Excel Spreadsheet (Note: The following instructions are for Excel 2003. They may have to be adjusted for newer versions of Excel.).

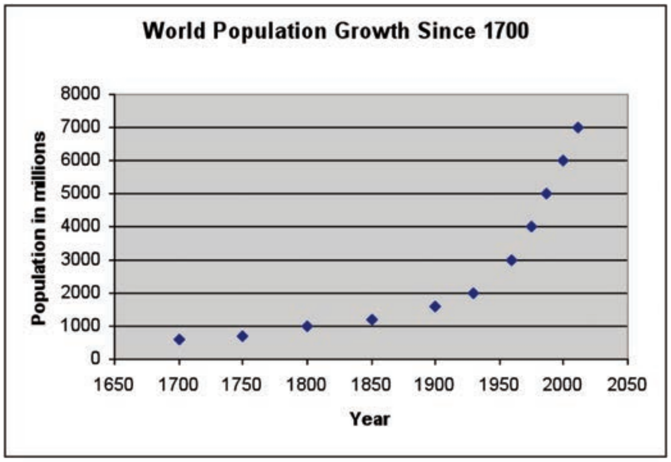
3. Enter the following data into the spreadsheet. This data shows the population growth from 1700 to 2011.

Year	World Population (in millions)
1700	610
1750	700
1800	1,000
1850	1,200
1900	1,600
1927	2,000
1960	3,000
1974	4,000
1987	5,000
1999	6,000
2011	7,000

4. Highlight the data, and then go to the "Insert" drop-down menu and select "Chart." Select "Scatter Graph" or "XY/Scatter" and create a graph of points with year on the x-axis (horizontal) and world population on the y-axis. Create the appropriate labels for the axes and the chart title.

5. To zoom-in to the correct years, change the scale of the x-axis so that it goes from 1700-2100. Right click on the x-axis labels and choose "Format axis." Change the minimum to 1700 and the maximum to 2100.

   Tell students that population growth has been a very recent phenomenon, starting around 1700 and accelerating from there, as the graph shows on the following page.



6. One way to model data in a scatter plot is to use lines of best fit, or trendlines. Add different types of trendlines by right-clicking on the data points and choosing “Add Trendline.” An option comes up that allows you to choose a Linear, Logarithmic, Polynomial, Power, Exponential, or Moving Average trendline. Because population grows by a percentage each year, by definition, it is an exponential equation, so click that choice.

   If you have gone over R2 values or correlation coefficients with your students, you may want to incorporate them into the simulation. You can place the R2 value on the graph after you add a trendline by right-clicking on the trendline and selecting “Format Trendline.” In the pop-up menu, select the “Options” tab and check the box that says “Display R-squared value on chart.” You can also display the graph’s equation on the chart by selecting that option.

7. Ask the students to write down this equation on their Student Worksheet and answer the questions that follow about projected population growth based on this estimate. They will need calculators because they are dealing with very large numbers.

Graphing Growth Student Worksheet Answers

1. y = 0.001e0.0077x
2. a. 7.17 billion
   b. 10.5 billion
   c. This estimate is lower than the United Nations’. Growth rates continually change with changes in demographics and society (fertility rates, life expectancy, etc.) At present, demographers expect world population to grow to 10.1 billion by 2100.
3. Answers will vary.

Name: ______________________ Date: ______________________

Calculating 7 Billion

Student Worksheet: Measuring a Million - Group 1

Problem:

a) What would be the length of a million people holding hands?
b) The length of a billion people holding hands?

Process:

1. I would estimate that a million people holding hands would stretch _______ miles. (*Are the arms stretched out or down?)*

2. The average estimate for my group is ______ miles.

3. Measure the length of 4 people standing holding hands.
   a) The length is _______ cm, or _________ meters.
   b) The ratio of people to length is ______ : ______.

   $$\frac{\text{4 people}}{\textit{Answer to 3a in meters}} = \frac{\text{1,000,000 people}}{x \text{ meters}}$$

   Cross multiply to solve for *x*. ___________ meters.

4. How many kilometers would the line of one million people be? ___________ km. How many miles long is this (1 mile - 1.6km)? ___________ miles.

5. Look at a map of your state or country. Can you find a distance between two cities that is equiva-
lent to the length of a million people holding hands? ___________

6. How many miles long would a line of one billion people holding hands be? ___________

7. The circumference of the Earth is roughly 25,000 miles. A line of a billion people holding hands would circle the globe how many times? ___________

8. Is this number greater than or less than what you and your group were expecting? Why do you think you over/under estimated?

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Calculating 7 Billion

Student Worksheet: Measuring a Million - Group 2

Problem:

a) How many people standing on each others' heads would it take to stack people to the moon?

b) How tall would 1 billion people be if they were stacked in this way?

Process:

1. I would estimate that ________________ people would stack to the moon.

2. The average estimate for my group is ________________ people.

3. Start by finding the average height of the people in your group. Have each group member convert their height into inches (for example, 5'2" is equal to 62 inches) and determine the aver age height of your group. ________________ in.

4. Now convert this average into centimeters (Remember: 1 in = 2.54 cm). On average, our group is ________________ cm tall.

5. How tall is the average human in kilometers? (Remember that 100 cm =1 m and 1000 m = 1 km) ________________ km

6. On average, the distance to the moon is 382,500 km. Knowing this, determine how many humans standing on top of each other it would take to stack humans to the moon.

$$\frac{1 \text{ person}}{\textit{Answer to \#5} \text{ km}} = \frac{x \text{ people}}{382{,}500 \text{ km}}$$

   Cross multiply to solve for x. ________________ humans

7. Is this greater or less than your estimate? Why do you think you over/under estimated?

8. To answer part B, multiply your answer to #5 by 7 billion. How many times could you go to the moon and back if you stacked 7 billion people on their heads?

9. Is this number greater than or less than what you and your group were expecting? Why do you think you over/under estimated?

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Calculating 7 Billion

Student Worksheet Measuring a Million - Group 3

Problem:

a)How many sheets of copy paper could fit in the classroom?
b)How many classrooms would you need to fit one billion sheets of paper?

Process:

1. I would estimate that __________ sheets of copy paper would be able to fit in the classroom.

2. The average estimate for my group is __________ sheets of paper.

3. Use a meter stick to measure the length, width, and height of a ream 500 sheets as it is packaged) of copy paper (100 cm = 1 m).
   Length: _________ m
   Width: _________ m
   Height: _________ m

   Volume = Length x Width x Height= ________________ m^3

4. Calculate the volume of one million sheets of paper.
   $$\frac{\text{Answer to \#3}}{500} = \frac{\text{x meters}^3}{1{,}000{,}000}$$
   Cross multiply to solve for x. ________________ m^3

5. Calculate the volume of one billion sheets of paper.
   $$\frac{\text{Answer to \#3}}{500} = \frac{\text{x meters}^3}{1{,}000{,}000{,}000}$$
   Cross multiply to solve for x. ________________ m^3

6. Now that you know the volume of a billion sheets of paper, you can begin to solve part B of the question. Use the meter stick to measure the length, width, and height of the classroom.
   Length: _______ m
   Width: _______ m
   Height: _______ m

7. Calculate the volume. ________________ m^3

8. Calculate the number of classrooms needed to fit one billion sheets of paper.
   $\dfrac{\textit{Answer to \#3}}{\textit{Answer to \#7}}$ = Number of classrooms needed ________________

9. Is this number greater or less than what you and your group were expecting? Why do you think you over/under estimated?

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Calculating 7 Billion

Student Worksheet: Measuring a Million - Group 4

Problem:

a) How many boxes of crayons (a 24-pack) could fit in the classroom?

b) How many classrooms would you need to fit one billion crayon boxes?

Process:

1. I would estimate that ___________ boxes of crayons would be able to fit in the classroom.

2. The average estimate for my group is __________boxes of crayons.

3. Use a meter stick to measure the length, width, and height of the box of crayons in cm.
   Length: _________ cm
   Width: _________ cm
   Height: _________cm

4. Change the values into meters. (100 cm = 1 m)
   Calculate the volume of the box of crayons using the following formula.

   Volume = Length (m) x Width (m) x Height (m) = ______________ m^3

5. Use the meter stick to measure the length, width, and height of the classroom.
   Length: _______ m
   Width: _______ m
   Height: _______ m

6. Calculate the volume of the classroom. ________________ m^3

7. Calculate the number of crayon boxes that can fit in the classroom.
   $\frac{\text{Answer to \#6}}{\text{Answer to \#4}}$ = Number of crayon boxes _________________

8. Is this number greater than or less than what you and your group were expecting? Why do you think you over/under estimated?

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Calculating 7 Billion

Student Worksheet Measuring a Million - Group 5

Problem:

a) Imagine a crowd of a million people. How big a field would you need to contain them all?

b) How much land would you need to hold a billion people?

Process:

1. If an acre is about the size of a football field, you would probably need ________ acres to contain one million people. (Estimate)

2. The average estimate for my group is __________ acres.

3. Have four students in your group stand together in a cluster. Mark the space about them with chalk on the floor and measure the area of the space in square meters.

4. Set up a ratio of people to area to find how many square meters (m^2) are needed for a million people.

   $$\frac{4 \text{ people}}{\_\_\_\ m^2} = \frac{1{,}000{,}000 \text{ people}}{x\ m^2}$$

   Cross multiply to solve for x. _________________m^2

5. If 1 acre = 4,047 square meters (m^2), about how big a field would you need to contain one million people? _________ acres. What would this be in square kilometers (Note: 1 km^2 = 247 acres)? Can you think of an area in your community that is about this size?

6. How many acres of land would you need to contain one billion people? _________ acres. What would this be in square kilometers (km^2)? ________ Can you think of a state of country that is about this size? If not, check an atlas for some ideas.

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Calculating 7 Billion

Student Worksheet: Napkin Fold

# of times napkin is folded	# of napkin layers
0	1
1	2
2	4
3	8
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
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Name: ______________________________ Date:__________________________

Calculating 7 Billion

Student Worksheet: Graphing Growth

1. What is the equation of the exponential trend line on the graph of world population from 1700 to 2011? ______________________________

2. The following questions ask you to extrapolate this curve into the future to make projections about where population might head.

   a. Based on this curve, what will our population be in 2050 (Hint: Solve for y with x = 2050)?

   b. Based on this curve, what will our population be in 2100?

   c. According to the United Nations, the world population is projected to be around 9.3 billion in 2050.

   Is your calculated estimate greater than or less than the UN's? Why do you think that this discrepancy exists?

3. As population grows, land, food and water become limited. How do you think this will impact the future trend of population growth? Draw a sketch of what you think the graph will look like in the future.

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